Scalar-isoscalar mesons in pion production at threshold

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We present the contributions of the non-linear chiral scalar field S to cross sections of $pp \rightarrow d\pi^+$, $pp \rightarrow pn\pi^+$ and $pp \rightarrow pp\pi^0$ reactions at threshold. We compare our results with the meson σ contribution from the σ-linear model. We show that the chiral scalar field S is almost 5 times bigger than the other contributions, which indicates that the two-pion exchange dynamics embebbed in the S field is the most important term for the pion production at threshold.

The 90's decade presented the advent of new accelerators and very precise detecting systems, which led to good and accurate data for $np \rightarrow d\pi^0$ [1], $pp \rightarrow pp\pi^0$ [2,3], $pp \rightarrow d\pi^+$ [4,5], and $pp \rightarrow pn\pi^+$ [6,7] reactions near threshold. Before these new data, nuclear S-wave pion production was usually described by a single nucleon (impulse approximation term) and a pion rescattering mechanism (seagull term) [8]. However, these new data proved that the impulse approximation and the pion rescattering term were not enough to take into account the magnitude of the new cross sections [9], underestimating them by a factor of 5. To fix this problem, many authors suggested the use of short range dynamics [10–12] through the exchange of heavy mesons like σ and ω, which proved that a scalar-isoscalar meson has a critical role in π^0 production. However, this procedure presented a lot of theoretical uncertainty embebbed in the coupling constants, which motivates Cohen *et al.* [13] to use Chiral Perturbation Theory (χ_{PT}) to study the π^0 production. The main idea was to organize the several potentially significant mechanisms of pion production. They adapted the power counting and estimated leading and next-to-leading contributions, but the final result was again a factor of 5 below data. Other χ_{PT} calculations were performed, stressing the importance of (a) the rescattering term [14] and (b) the effect of loops [14–17]. The main problem in these evaluations is the large number of contributions, especially in the loop case, which turns the result to have some theoretical uncertainty. Therefore it would be very useful if we could take into account the intermediate range contribution of the two-pion loop in some effective and easier manner. The most well known procedure to do that is to represent the pionic loop by an effective scalar-isoscalar meson, as done by several authors regarding the NN interaction [18,19] and, in this paper, we are looking close at contribution of such scalar-isoscalar meson exchange between nucleons to the cross section of pion production at threshold. Such scalar-isoscalar meson has been used to improve the contribution of impulse and rescattering terms [10–12].

There are two models for scalar-isoscalar meson: σ-linear model and a scalar field S implemented in the framework of nonlinear Lagrangian which is chiral invariant and appears naturally when nonlinear fields are obtained from linear ones [20]. Comparison between chiral S and σ-linear in three-body forces [21] shows a favorable result to the former.

In the framework of $\sigma-$linear model the dynamics comes from the following Lagrangian

$$L_\sigma = -g\bar{N}\left(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5\right)N\,, \tag{1}$$

where $\vec{\pi}$ is the pion field, N is the nucleon field that transforms linearly. The non-linear Lagrangian for the scalar-isoscalar meson reads

$$L_S^{PV} = \frac{g}{2m}\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot D^\mu\vec{\phi} - g_S S\bar{\psi}\psi\,, \tag{2}$$

and equivalent results to (2) can be obtained from the following pseudoscalar form

$$L_S^{PS} = -\left(g + \frac{g_S}{f_\pi}S\right)\bar{N}\left(f\left(\vec{\phi}\right) + i\vec{\tau}\cdot\vec{\phi}\gamma_5\right)N + \cdots\,, \tag{3}$$

where f_π, g and g_S are, respectively, pion decay constant, πN coupling constant and SN coupling constant, $\vec{\phi}$ is the pion field in nonlinear realization and $f\left(\vec{\phi}\right) = \sqrt{f_\pi^2 - \vec{\phi}^2}$; in "..." we subsume all other possible interactions that contribute to the pion production, but are not required here since they are of lower magnitude. The pion-nucleon

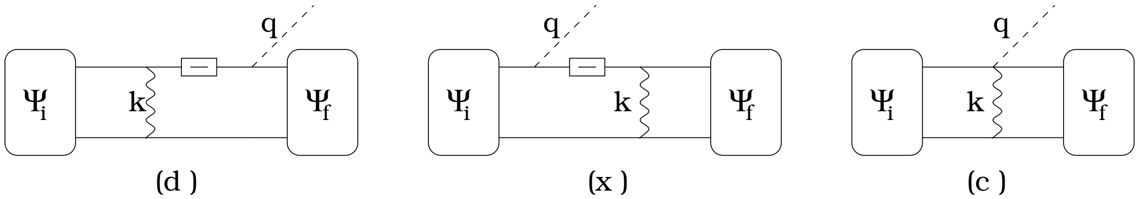

FIG. 1. Diagrams with the chiral scalar S (wavy line) that contribute to the pion production (dashed line). The minus sign in the nucleon propagator (full line) indicates that the positive energy states were removed. The labels means (d) for direct, (x) for crossed and (c) for contact.

decay constant f_π relates to g by the Goldberger-Treimann (GT) relation, $g\, f_\pi = g_A\, m$, where g_A is the axial pion-nucleon coupling constant and m the nucleon mass. This is a phenomenological requirement since the Lagrangians in Eqs. (1) and (3) predicts $g_A = 1$. The Lagrangian form in Eq. (3)is more suitable to our discussion because it is similar to (1) and shows explicitly an additional $S\pi NN$ vertex which is absent in (1). Equivalence between pseudoscalar and pseudovector forms was verified explicitly in the case of TPEP [22] and in πN form factor [23]. More details of chiral scalar can be found in Refs. [21,23]. It is worth noting that S and σ fields behave differently under chiral transformation. In the non-linear realization of chiral symmetry, the axial transformation of S results in : $\delta^A S = 0$, while, in the linear realization, one has $\delta^A \sigma = \vec{\beta}\cdot\vec{\pi}$, where $\vec{\beta}$ is an arbitrary isovector.

Both Lagrangians in Eqs. (1) and (3) have an isoscalar-nucleon and a pion-nucleon interaction that give rise to the first two diagram of Fig. 1d,x for pion production process. However, the chiral S Lagrangian has an extra isoscalar-pion-nucleon interaction. This extra interaction vertex give raise to the contact diagram depicted Fig. 1c, which represents a genuine difference between σ-linear and chiral scalar S lagrangians.

As the first two diagrams of Fig. 1 are common to both Lagrangians, we set a parameter λ to turn on ($\lambda = 1$) or off ($\lambda = 0$) the contributions of contact term and write

$$T(\lambda) = -igg_S^2\tau_c \left\{ \bar{u}(\mathbf{p}') \left[\frac{-\not{q}}{p_d^2 - m^2} + \frac{-\not{q}}{p_x^2 - m^2} + \frac{\lambda}{g\, f_\pi} \right] \gamma_5 u(\mathbf{p}) \right\}^{(1)} \frac{1}{k^2 - m_s^2} \left[\bar{u}(\mathbf{p}')\, u(\mathbf{p})\right]^{(2)} + (2 \to 1)\ , \tag{4}$$

where τ_c is the isospin operator for the emitted pion, $\bar{u}(\mathbf{p}')$ and $u(\mathbf{p})$ are the spinors of nucleons with momentum $\mathbf{p}'$ and $\mathbf{p}$ respectively, $k = p_2' - p_2$, m and m_S are the nucleon and scalar masses, the labels (1) and (2) denote nucleon 1 and 2, and the labels d and x means direct and crossed diagrams.

Following the procedure applied in [21], we extract the irreducible amplitude in order to avoid double counting due to positive frequency propagation of nucleons; the result is the following proper amplitude for the process $\pi N_j N_k$

$$\begin{aligned} T_{\pi NN}(\lambda) &= -igg_S^2 \left\{ \tau_c \bar{u}(\mathbf{p}') \left[\frac{\not{q}}{2E_d\,(p_d^0 + E_d)} + \frac{\not{q}}{2E_x\,(p_x^0 + E_x)} - \left(\frac{1}{2E_d} - \frac{1}{2E_x} \right)\gamma^0 + \frac{\lambda}{gf_\pi} \right] \gamma_5 u(\mathbf{p}) \right\}^{(1)} \\ &\quad \times \frac{1}{k^2 - m_s^2} \left[\bar{u}(\mathbf{p}')\, u(\mathbf{p})\right]^{(2)} + (2 \to 1)\,, \end{aligned} \tag{5}$$

where $E_{d,x} = \sqrt{\mathbf{p}_{d,x}^2 + m^2}$ is the on mass-shell nucleon energy.

In order to obtain the production kernel we adopt the center of mass reference and take the non-relativistic limit of the amplitude, Eq. (5), keeping terms up to order ω/m where $\omega = E' - E$. We obtain the following relations:

$$\frac{1}{2E_d} - \frac{1}{2E_x} \sim \frac{\mathbf{p}^2}{m^3}, \tag{6}$$

$$\frac{1}{2E_d\,(p_d^0 + E_d)} + \frac{1}{2E_x\,(p_x^0 + E_x)} \sim \frac{1}{2m^2}, \tag{7}$$

$$\bar{u}(\mathbf{p}')\, u(\mathbf{p}) \to \frac{(\bar{E} + m)}{2m} \left[1 - \frac{\sigma\cdot\mathbf{p}'\sigma\cdot\mathbf{p}}{(\bar{E} + m)^2} \right], \tag{8}$$

$$\bar{u}(\mathbf{p}')\, \gamma_5 u(\mathbf{p}) \to -\frac{1}{2m} \left[\sigma\cdot(\mathbf{p}' - \mathbf{p}) - \frac{\omega}{2\,(\bar{E} + m)} \sigma\cdot(\mathbf{p}' + \mathbf{p}) \right], \tag{9}$$

$$\bar{u}(\mathbf{p}')\,\gamma^0\gamma_5 u(\mathbf{p}) \rightarrow \frac{1}{2m}\left[\sigma\cdot(\mathbf{p}'+\mathbf{p}) - \frac{\omega}{2(\bar{E}+m)}\sigma\cdot(\mathbf{p}'-\mathbf{p})\right], \tag{10}$$

$$\bar{u}(\mathbf{p}')\,\not{k}\gamma_5 u(\mathbf{p}) \rightarrow \frac{(\bar{E}+m)}{2m}\chi^\dagger\left[\frac{\omega}{(\bar{E}+m)}\sigma\cdot(\mathbf{p}+\mathbf{p}') - \sigma\cdot\mathbf{k}\right], \tag{11}$$

$$\frac{1}{q^2-m_s^2} \cong -\frac{1}{\mathbf{q}^2+m_s^2}, \tag{12}$$

where $\bar{E} = (E'+E)$. Using these results, and keeping only terms of the order $\mathbf{p}/m$, we get the non-relativistic amplitude:

$$t_{\pi NN}(\lambda) = -ig\frac{\omega_q}{2m}\frac{1}{2m^2}\,\frac{g_S^2}{\mathbf{k}^2+m_S^2}\left\{\tau_c^{(1)}\left[\left(1+\frac{\lambda}{2}\right)\sigma^{(1)}\cdot(\mathbf{p}+\mathbf{p}') + \lambda\frac{2m}{\omega_q}\sigma^{(1)}\cdot(\mathbf{p}'-\mathbf{p})\right]\right\} + (1\leftrightarrow 2)\,, \tag{13}$$

where μ is the pion mass and $\omega_q = \sqrt{\mathbf{q}^2+\mu^2}$ is the on-shell emitted pion energy with trimomentum $\mathbf{q}$ in the center of mass reference; $\mathbf{k} = \mathbf{p}-\mathbf{p}'$ is the momentum transferred; $\sigma^{(i)} = \langle\chi^i|\sigma|\chi^i\rangle$, $\tau_c^{(1)} = \langle\eta^i|\tau_c|\eta^i\rangle$, $|\chi^i\rangle$ and $|\eta^i\rangle$ are spin and isospin states of nucleon i respectively. At this point it is possible to figure out the main difference between chiral scalar S and σ-linear. The pure σ contribution gives (see Ref. [13], Eq. (16)):

$$t_{\pi NN}^{\sigma} = -ig\frac{\omega_q}{2m}\frac{1}{2m^2}\,\frac{g_S^2}{\mathbf{k}^2+m_S^2}\left\{\tau_c^{(1)}\sigma^{(1)}\cdot(\mathbf{p}+\mathbf{p}')\right\} + (1\leftrightarrow 2)\ . \tag{14}$$

where we made use of GT relation. We can see that the contribution of $S\ (\lambda=1)$ adds 50% in the term proportional to $(\mathbf{p}+\mathbf{p}')$ and a new factor of $(2m/\omega_q)$ which multiplies the momentum exchanged. This new term will generate a derivative of the Yukawa function in the coordinate space, which is the same kind of contribution given by the Weinberg-Tomosawa term [8]. We can see the consequence of this new term at threshold ($\omega_q \approx \mu$, $\frac{2m}{\mu} \approx 13$) in Fig. 2, where we show the dominant role of chiral scalar for the reaction $pp \to d\pi^+$. We set $g_S = g_\sigma = 8.7171$ and $m_S = m_\sigma = 550$ MeV in order to compare the results of both models. These parameters are obtained from table A.3 of Ref. [19], however, as S is an effective particle both g_S and m_S are free parameters. In principle one can extract the mass of the chiral scalar by a careful analysis of the two pion exchange role in the NN potential. This work is now in progress [24].

The non-relativistic approach to evaluate the cross section uses eigenstates $|NN\rangle$ of realistic NN potentials to split the correlations between nucleons from interactions which lead to pion production. Cohen *et al.* [13] showed that the difference between the several realistic potentials available in the literature are minimal, except for the Δ contribution. Therefore we choose here to work with the Argonne v18 Potential [25]. Thus, the cross section reads

$$\sigma \propto \langle NN|\ |t_S|^2\ |NN\rangle\ , \tag{15}$$

where t_S is the pion production amplitude in the configuration space due to the chiral scalar. In order to calculate t_S, we follow Ref. [11] by including the effects of form factors defined by the following replacement:

$$g_S \rightarrow g_S\frac{\Lambda_S^2-m_S^2}{\Lambda_S^2-\mathbf{k}^2}, \tag{16}$$

where $\mathbf{k}$ is the transferred momentum and $\Lambda_S = 2000$ MeV is the cutoff mass, listed in Table A.3 of Ref. [19]. The final amplitude in the coordinate space at threshold reads:

$$t_S(\lambda) = -i\frac{g}{2m}\frac{g_S^2}{4\pi}\frac{\omega}{2m}\left\{\frac{1}{m}F_1(r,\Lambda_S)\,\boldsymbol{\Sigma}_c\cdot 2\mathbf{p} - \frac{1}{m}i\boldsymbol{\Sigma}_c\cdot\hat{\mathbf{r}}F_2(r,\Lambda_S) + \frac{\lambda}{2m}F_1(r,\Lambda)\,\boldsymbol{\Sigma}_c\cdot 2\mathbf{p} - \frac{\lambda}{2m}\left(1-\frac{4m}{\omega}\right)i\boldsymbol{\Sigma}_c\cdot\hat{\mathbf{r}}F_2(r,\Lambda_S)\right\}, \tag{17}$$

where

$$F_1(r,\Lambda) = \left(\frac{e^{-m_s r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{1}{2\Lambda}\left(\Lambda^2-m_S^2\right)e^{-\Lambda r}\right), \tag{18}$$

$$F_2(r,\Lambda) = \left(-(1+m_s r)\frac{e^{-m_s r}}{r^2} + (1+\Lambda r)\frac{e^{-\Lambda r}}{r^2} + \frac{1}{2}\left(\Lambda^2-m_S^2\right)e^{-\Lambda r}\right), \tag{19}$$

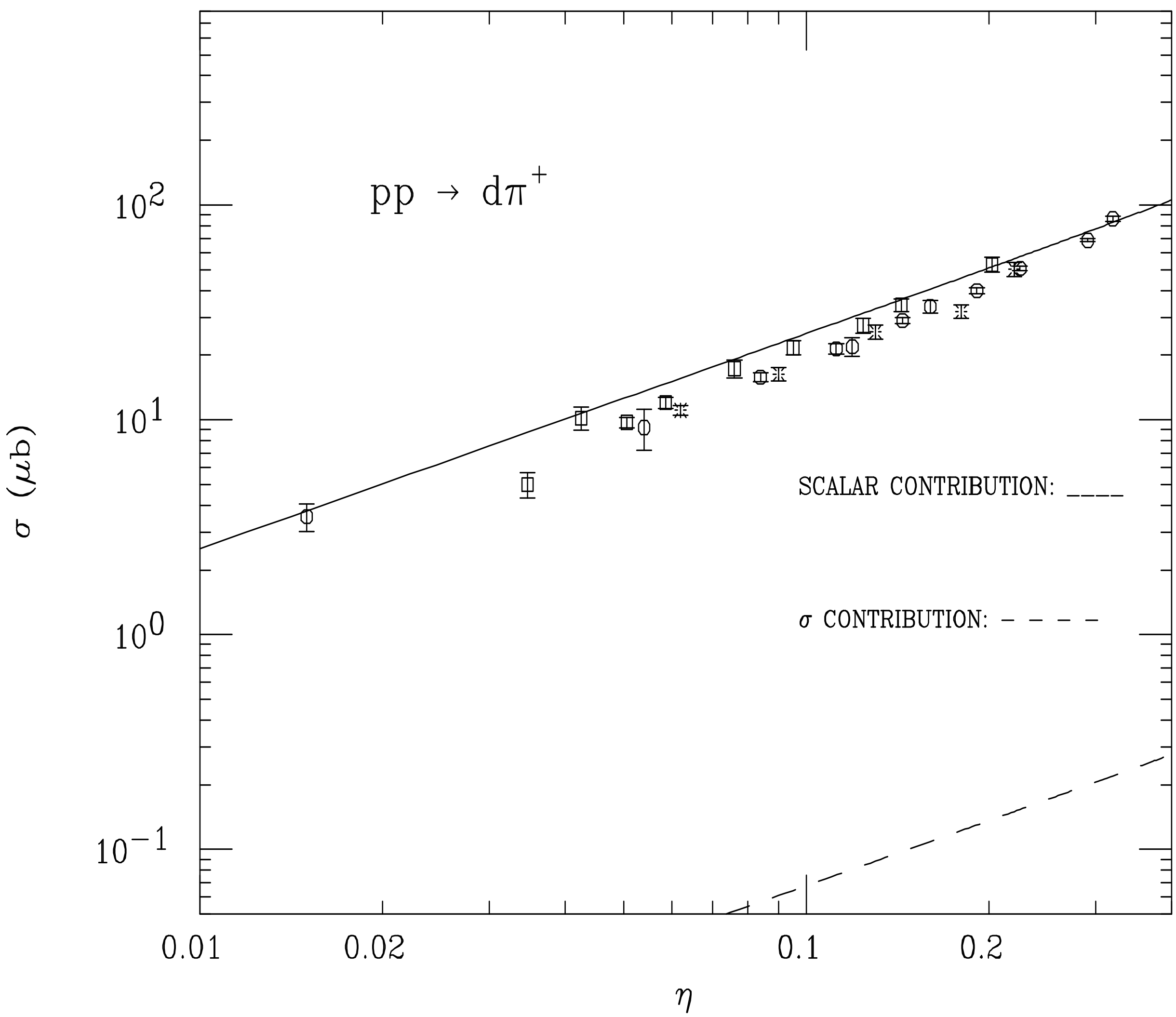

FIG. 2. Cross-section for the deuteron final state as function of η. The graph show the contributions of chiral scalar (full line) and the σ meson (dashed line), where the potential used is Argonne v18. Data are from TRIUMF (squares) [1], COSY (diamonds) [4] and IUCF(circles) [5]. In this comparison, our results are obtained in the same fashion as done in Fig. 3.

TABLE I. Preliminary σ/η results for the reaction $pp \to d\pi^+$ for $\eta = 0.01$, where the abbreviations read as: WT: Weinberg-Tomosawa, IA: impulse approximation, Δ: delta resonance contribution, GC: Galilean correction to WT, σ: σ linear meson contribution, and SEA: $\pi\pi NN$ isoscalar contribution. All these terms except the S contribution can be seen in Refs. [13,26].

Contribution	Scalar	WT	GC	IA	Δ	σ	SEA
σ/η (μb)	262	110	3.72	0.39	1.24	0.18	0.64

and $\mathbf{\Sigma}_c \equiv \left(\tau_c^{(1)}\sigma^{(1)} - \tau_c^{(2)}\sigma^{(2)}\right)$.

The next step is the evaluation of the matrix element of the operator in Eq. (17) between the initial and final states for the reactions $pp \to d\pi^+$, $pp \to pn\pi^+$ and $pp \to pp\pi^0$. We limit ourselves to the threshold region, where the two final state nucleons are in a strong attractive $L_{NN} = 0$ state, and we can consider the pion angular momentum $\ell_\pi = 0$. The last step is to square the amplitudes and integrate over the phase space avaliable. These calculations are straightforward and can be found in [13] for the $pp \to pp\pi^0$ reaction and in [26] for the other two reactions.

We show our results in Fig. 3, where we plot the cross sections for the reactions $pp \to pp\pi^0$, $pp \to d\pi^+$, and $pp \to pn\pi^+$ as a function of the pion momenta $\eta = \frac{q}{\mu}$. The impressive agreement between the chiral S scalar contribution and the data reveals that this is the major contribution for the pion production at threshold. Since this chiral S simulates two pion exchanges, this result corroborates that the pion production reaction is a suitable framework to study the medium and short ranged dynamics of NN interaction and, in some sense, contributions due to long ranged terms have a minor role in these reactions. It is worth noting that this work do not intend to fit the data, but only show that we do not have the old factor of 5 below data anymore. A detailed study comparing this chiral S and long ranged terms will be shown elsewhere [27], but we present in Tab. I some preliminary results for the $pp \to d\pi^+$ cross section. We see in the table that the S contribution is predominant and is due to the $S\pi NN$ vertex term, as seen in the Fig. 1c. When we turn this contribution off ($\lambda = 0$ in Eq. (17)), the results of S and σ are identical. This means that the direct and crossed diagrams presented in both contributions are of order μ/m lower in comparison with the $S\pi NN$ term. In addition, the table shows that the contributions of IA, Δ and SEA are of order μ/m lower than the S term, and the WT term is an exception, due to the isospin coefficients.

Our results for the S contribution to the pion production have shown that the cross section for these reactions near threshold can be explained by taking into account the contribution of the medium-range three-pion contact interaction with the nucleon, here represented by the $S\pi NN$ vertex. In a χ_{PT} approach [13], this represents that the second order contributions, expressed mainly by the two-pion loops, are the most important contributions. It could be an indication of a connection between the energy threshold of a particular reaction and the dominant order of the power counting. Since the chiral S scalar simulates the two-pion exchange in the pion production, one important issue here is to check if the sum of all contributions of second order in χ_{PT} gives the same result obtained here. This is a herculean task, but some promising results were shown recently [17].

To conclude, we have shown that the cross section for the reactions $pp \to pp\pi^0$, $pp \to d\pi^+$, and $pp \to pn\pi^+$ near threshold can be explained by taking into account the contribution of the medium-range three-pion contact interaction with the nucleon, here represented by the $S\pi NN$ vertex, to the pion production amplitude; in a χ_{PT} approach, this fact means that the second order contribution, expressed mainly by the two-pion loops, is the most important contribution. We believe that should be a close connection between the energy threshold of a particular reaction and the dominant order of the power counting. This idea may be not new, but the present results can be understood using this different approach.

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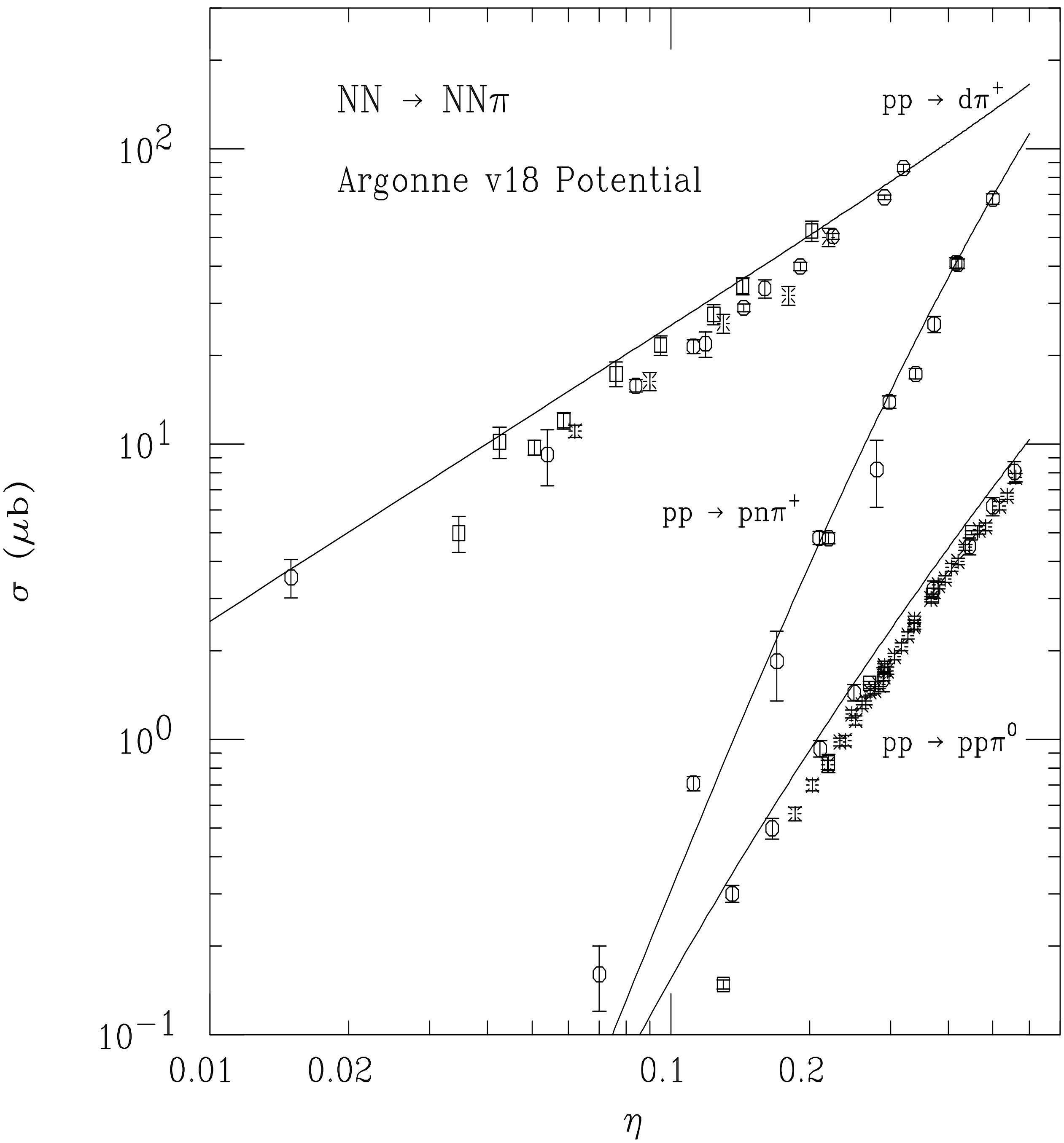

FIG. 3. Cross-section σ for the pion production as function of η. The lines shown the contribution of the chiral S scalar for the reactions $pp \to pp\pi^0$, $pp \to d\pi^+$, and $pp \to pn\pi^+$ using the Argonne V18 potential. The $pp \to pp\pi^0$ data are from [2] (circles) and [3] (crosses); the $pp \to d\pi^+$ data are from [1] (crosses), [4] (squares) and [5] (circles); the $pp \to pn\pi^+$ data are from [6,7] (circles).

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